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2008 J. Phys.: Condens. Matter 20 345228

(http://iopscience.iop.org/0953-8984/20/34/345228)

J. Phys.: Condens. Matter **20** (2008) 345228 (13pp) doi:10.1088/0953-8984/20/34/345228

Magnetotransport of Dirac fermions in graphene in the presence of spin–orbit interactions

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Received 4 June 2008, in final form 18 July 2008
Published 7 August 2008
Online at stacks.iop.org/JPhysCM/20/345228

Abstract
We study theoretically the quantum transport in graphene while accounting for spin–orbit interactions (SOIs). Our method is based on the Schwinger proper-time Green's function and a decomposition over Landau level poles and the Kubo formula. Analytical expressions for both the longitudinal and the Hall conductivities are derived and given explicitly. We find, when the Rashba SOI is taken into account, the Shubnikov–de Haas (SdH) oscillation peaks of the longitudinal conductivity versus the chemical potential are split, while the SdH oscillation of the longitudinal conductivity versus a external magnetic field exhibits a beating pattern. The temperature dependence of the longitudinal conductivity becomes non-monotonic for nonzero field away from half-filling. The Rashba SOI tends to suppress the quantum Hall effect in graphene.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

A number of fascinating physical properties have been detected or predicted to occur in graphene [1–9], such as the anomalous quantized Hall effect, the absence of the weak localization and other conductivity features [3]. In addition to dissipative transport supercurrent transport has also been observed [4]. Experimentally, it is now possible to produce controllably a few layers or a single monolayer of graphene [10–13]. Graphene is also shown to be a promising candidate for applications in spintronics devices that exploit the effects inherent to SOIs [14, 15]. Theoretically, Kane and Mele inspected the effect of SOIs in graphene [16, 17] and find the spin Hall conductivity to be quantized in the absence of a magnetic field because of a gap produced by SOIs. Depending on the relative strength of the intrinsic and Rashba SOIs, it is further shown that the spin Hall conductivity can be zero or nonzero. Kane and Mele gave an estimate of the SOI scale which was re-examined by Yao *et al* and Min *et al* [18, 19] who provided some explicit expressions of the SOIs in graphene.

The purpose of this work is to investigate the transport of Dirac fermions in graphene with a special focus on the effects of SOIs on the transport using the estimation of Yao *et al* and Min *et al* for the SOIs in graphene. Using the Schwinger proper-time method [20], a decomposition over Landau level poles [21, 22] and the Kubo formula [23], we provide analytical expressions for both the longitudinal and the Hall conductivities. We find in the presence of Rashba SOIs the longitudinal conductivity as a function of the chemical potential deviates from the linear relation at zero magnetic field. For nonzero magnetic field, the SdH oscillations are observed and the oscillation peaks in the longitudinal conductivity versus the chemical potential are split when the Rashba SOI is applied, while the oscillation in the longitudinal conductivity versus the magnetic field exhibits a beating pattern. When graphene is away from half-filling, the longitudinal conductivity shows a non-monotonic feature at finite field strengths. It is also shown that the Rashba SOI tends to suppress the quantum Hall effect in graphene.

This paper is organized as follows. In section 2, a model of a single layer of graphite (graphene) with SOIs is presented. In section 3, we derive the analytical expressions for both the longitudinal conductivity and the Hall conductivity including the limits of these expressions at a zero field. In section 4, the corresponding numerical results are presented and discussed. We conclude in section 5 with a summary and appendices containing some involved algebraic manipulations.

2. Formal framework

Graphene is a flat monolayer of carbon atoms tightly packed into the honeycomb lattice. At low energy, it can be described by a $(2+1)$-dimensional relativistic field theory model. When the SOIs are included, the Lagrangian density of the system is given by

$$\mathcal{L} = \hbar v_F \overline{\Psi}(\mathrm{i}\hat{D} + H_s)\Psi, \tag{1}$$

where $\Psi = (\Psi_K, \Psi_{K'})$ is the eight-component Dirac spinors with $\Psi_{K(K')} = (\Psi_{A\uparrow}, \Psi_{A\downarrow}, \Psi_{B\uparrow}, \Psi_{B\downarrow})$ which describes the spin-resolved states residing on the atoms of the A, or B sublattices at the momentum $K(K')$, $\hat{D} = \gamma^\mu(\partial_\mu - \mathrm{i}eA_\mu)$ with γ^μ ($\mu = 0, 1, 2$) being the (4×4) γ matrices, $\gamma^\mu = \sigma_3 \otimes (\sigma_3, \mathrm{i}\sigma_2, -\mathrm{i}\sigma_1)$, e is the electron charge, v_F is the Fermi velocity, the external magnetic field $\mathbf{B} = \nabla\times\mathbf{A}$ is applied perpendicular to the x–y plane and the corresponding vector potential is taken in the symmetric gauge $\mathbf{A} = (-By/2, Bx/2)$. In equation (1), H_s describes SOIs. It has the form [18, 19]

$$H_s = \lambda_{SO}(\gamma^0 - s_z) + \lambda_R(-\mathrm{i}\gamma^0\gamma^2 s_y + \gamma^2 s_x), \tag{2}$$

where λ_{SO} is the intrinsic SOI parameter, λ_R is the Rashba SOI parameter and $\mathbf{s}$ is a spin variable. For $B = 0$, the energy spectrum is given by

$$\begin{aligned}\varepsilon_1 &= \pm\sqrt{\mathbf{k}^2 + (\lambda_R - \lambda_{SO})^2} + \lambda_R - \lambda_{SO},\\ \varepsilon_2 &= \pm\sqrt{\mathbf{k}^2 + (\lambda_R + \lambda_{SO})^2} - \lambda_R - \lambda_{SO}.\end{aligned} \tag{3}$$

For $\lambda_{SO} > \lambda_R > 0$, the system includes an energy gap $2(\lambda_{SO} - \lambda_R)$. For $0 < \lambda_{SO} < \lambda_R$, the energy gap closes. The energy spectrum is essentially consistent with the result in [16]. This energy gap is due to the intrinsic SOI which violates time reversal symmetry and is related to a model introduced by Haldane as a realization of the parity anomaly in $(2+1)$-dimensional relativistic field theory [9].

The Green's function of Dirac fermions described by the Lagrangian (1) in an external magnetic field can be expressed as [20–22]

$$G(x,y) = (\mathrm{i}\hat{D} - H_s)_x \langle x| \frac{-\mathrm{i}}{H_s^2 + \hat{D}^2 - \mathrm{i}[\hat{D}, H_s]} |y\rangle. \tag{4}$$

Using the Schwinger proper-time approach [20], we obtain

$$G(x,y) = \exp\left(\mathrm{i}e\int_y^x A_\lambda \mathrm{d}z^\lambda\right)\widetilde{G}(x-y), \tag{5}$$

$$\begin{aligned}\widetilde{G}(x) = \int_0^\infty \mathrm{d}s \frac{\mathrm{e}^{-\frac{\mathrm{i}\pi}{4}}}{8(\pi s)^{3/2}} \mathrm{e}^{-\frac{\mathrm{i}}{4s}x_\nu C^{\nu\mu}x_\mu} &\times \left[\frac{1}{2s}\gamma^\mu C_{\mu\nu}x^\nu - \frac{1}{2}(e\gamma^1 Bx_2 - e\gamma^2 Bx_1)\right.\\ &\left. + \lambda_{SO}(1-\gamma^0 s_z)\right]\frac{eBs}{\sin(eBs)} \mathrm{e}^{\mathrm{i}(\frac{1}{2}e\sigma F - \Delta - B_\mu^2)s},\end{aligned} \tag{6}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $C^{\mu\nu} = g^{\mu\nu} + \frac{(F^2)^{\mu\nu}}{B^2}[1 - eBs\cot(eBs)]$ with $g^{\mu\nu} = \mathrm{diag}(1,-1,-1)$, and

$$B_\mu = \mathrm{i}2\lambda_{SO}\sigma^{\mu 0}s_z + 2\lambda_R(\sigma^{\mu 1}s_y - \delta_{\mu 0}\gamma^1 s_x - \delta_{\mu 1}\gamma^0 s_x), \tag{7}$$

$$\Delta = 2(\lambda_{SO}^2 + \lambda_R^2)(1-\gamma^0 s_z) + 4\lambda_{SO}\lambda_R(\mathrm{i}\gamma^1 s_y - \gamma^0\gamma^1 s_x) \tag{8}$$

with $\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$. It is clear that the symmetric gauge sets the factor $\exp(\mathrm{i}e\int_y^x A_\lambda \mathrm{d}z^\lambda) = 1$ in equation (5). Therefore, equation (5) becomes a function of the difference $x-y$ only. Using the expansions of the exponential function e^x, one can show that

$$\begin{aligned}\mathrm{e}^{\mathrm{i}(\frac{1}{2}e\sigma F - \Delta - B_\mu^2)s} &= \mathrm{e}^{-\mathrm{i}(10\lambda_{SO}^2 + 2\lambda_R^2)s}[\cos(\zeta s) + \mathrm{i}\gamma^0 s_z \sin(\zeta s)]\\ &\times \left\{\cos(eBs) + \gamma^1\gamma^2\sin(eBs) + \frac{1}{2}(1-\gamma^0 s_z)\right.\\ &\times [\cos(\xi s) - \cos(eBs)] - \mathrm{i}\frac{12\lambda_{SO}\lambda_R\sin(\xi s)}{\xi}\\ &\times (\mathrm{i}\gamma^1 s_y - \gamma^0\gamma^1 s_x) + \frac{1}{2}(1-\gamma^0 s_z)\gamma^1\gamma^2\\ &\times \left.\left[\frac{eB}{\xi}\sin(\xi s) - \sin(eBs)\right]\right\},\end{aligned} \tag{9}$$

where $\zeta = 2\lambda_{SO}^2 - 6\lambda_R^2$, $\xi = \sqrt{(24\lambda_{SO}\lambda_R)^2 + (eB)^2}$. Substituting equation (9) into (5), applying a Fourier transform in the Matsubara representation and using the decomposition method over Landau level poles [21, 22], we can derive the expression

$$G(\mathrm{i}\omega_m, \mathbf{k}) = G_1(\mathrm{i}\omega_m, \mathbf{k}) + G_2(\mathrm{i}\omega_m, \mathbf{k}) + G_3(\mathrm{i}\omega_m, \mathbf{k}), \tag{10}$$

after some straightforward but lengthy algebraic calculations. The expressions for $G_i(\mathrm{i}\omega_m, \mathbf{k})$ ($i = 1, 2, 3$) are quite involved and are provided in appendix A. The retarded and the advanced Green's functions are obtained by an analytic continuation, i.e. $G^{(R)}(\omega + \mathrm{i}0, \mathbf{k}) = G(\mathrm{i}\omega_m \to \omega + \mathrm{i}0, \mathbf{k})$ and $G^{(A)}(\omega - \mathrm{i}0, \mathbf{k}) = G(\mathrm{i}\omega_m \to \omega - \mathrm{i}0, \mathbf{k})$. The influence of impurities is accounted for phenomenologically by a constant scattering rate Γ. The Green's functions acquire the form

$$\begin{aligned}G^{(R,A)}(\omega, \mathbf{k}) &= G_1^{(R,A)}(\omega \pm \mathrm{i}\Gamma, \mathbf{k}) + G_2^{(R,A)}(\omega \pm \mathrm{i}\Gamma, \mathbf{k})\\ &+ G_3^{(R,A)}(\omega \pm \mathrm{i}\Gamma, \mathbf{k}).\end{aligned} \tag{11}$$

In general, the scattering rate $\Gamma(\omega) = -\mathrm{Im}\Sigma^R(\omega)$ is frequency-dependent and has to be determined self-consistently from the Schwinger–Dyson equations, as done in [24]. Here our focus is on the role of SOIs in transport. The exact form of interactions between impurities and electrons is not inspected.

3. Electronic conductivity

The Kubo formula for the frequency-dependent electrical conductivity as a linear response function to an external electric field is [23]

$$\sigma_{ij}(\Omega) = \frac{\mathrm{Im}\,\Pi^R_{ij}(\Omega + \mathrm{i}0)}{\Omega}, \tag{12}$$

where i, j are component indices and $\Pi^R_{ij}(\omega)$ is the retarded current–current correlation function obtained by an analytical continuation of the Matsubara function:

$$\Pi_{ij}(\mathrm{i}\omega_n) = \frac{1}{V}\int_0^\beta \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega_n\tau}\langle T_\tau J_i(\tau)J_j(0)\rangle, \qquad \omega_n = 2\pi nT. \tag{13}$$

V is the volume of the system, β is the inverse temperature and $J_i(\tau) = \int \mathrm{d}^2 r j_i(\tau,\mathbf{r})$ with $j_i = -ev_{\mathrm{F}}\overline{\Psi}(\tau,\mathbf{r})\gamma^i\Psi(\tau,\mathbf{r})$. Since the scattering rate Γ is a constant impurity scattering is isotropic in k space. We follow the argument that, for a small impurity density the conductivity is hardly affected by vertex corrections [25]. Neglecting the impurity vertex corrections, the calculations of the conductivity reduce to the evaluation of the bubble diagram. Equation (12) can then be rewritten as

$$\begin{aligned}\sigma_{ij}(\Omega) = \frac{e^2v_{\mathrm{F}}^2}{2\pi\Omega}\mathrm{Re}\int_{-\infty}^{\infty}\mathrm{d}\omega\int\frac{\mathrm{d}^2k}{(2\pi)^2}\,\mathrm{tr}\{[n_{\mathrm{F}}(\omega) - n_{\mathrm{F}}(\omega+\Omega)]\\ \times\gamma^i G^R(\omega+\Omega,\mathbf{k})\gamma^j G^A(\omega,\mathbf{k})\\ + n_{\mathrm{F}}(\omega+\Omega)\gamma^i G^A(\omega+\Omega,\mathbf{k})\gamma^j G^A(\omega,\mathbf{k})\\ - n_{\mathrm{F}}(\omega)\gamma^i G^R(\omega+\Omega,\mathbf{k})\gamma^j G^R(\omega,\mathbf{k})\},\end{aligned} \tag{14}$$

where $n_{\mathrm{F}}(\omega)$ is the Fermi distribution function. Substituting equation (11) into (14), we obtain the longitudinal conductivity as

$$\sigma_{xx} = \frac{2e^2v_{\mathrm{F}}^2|eB|}{\pi^2}\mathrm{Re}\int_{-\infty}^{\infty}\mathrm{d}\omega\frac{1}{4T\cosh^2\frac{\beta(\omega-\mu)}{2}}A_L(\omega). \tag{15}$$

The Hall conductivity is

$$\begin{aligned}\sigma_{xy} = -\frac{2e^2v_{\mathrm{F}}^2|eB|\mathrm{sgn}(eB)}{\pi^2}\\ \times\,\mathrm{Im}\int_{-\infty}^{\infty}\mathrm{d}\omega\frac{1}{4T\cosh^2\frac{\beta(\omega-\mu)}{2}}A_H(\omega).\end{aligned} \tag{16}$$

All quantities on the right-hand side of these equations are provided explicitly in appendix B. Equations (15) and (16) establish the fundamental basis for investigating the SOI effect on the quantum transport properties of the Dirac fermions in graphene.

In the limit of zero field, the Hall conductivity vanishes. For the longitudinal conductivity and for $\lambda_{\mathrm{SO}} = 0$, using the asymptotic expansions

$$\psi(z) = \ln z - \frac{1}{2z} - \frac{1}{12z^2} + \frac{1}{120z^4} + \mathrm{O}\left(\frac{1}{z^5}\right), \tag{17}$$

we find

$$\begin{aligned}A_L(\omega) = \left[\frac{(\omega^2-\Gamma^2)}{6\lambda_{\mathrm{R}}^2} - \frac{6\lambda_{\mathrm{R}}^2(\omega^2+\Gamma^2)}{(2\Gamma\omega)^2+(6\lambda_{\mathrm{R}}^2)^2}\right]\\ \times\,\ln\frac{(-4\lambda_{\mathrm{R}}^2-\omega^2+\Gamma^2)^2+(2\Gamma\omega)^2}{(8\lambda_{\mathrm{R}}^2-\omega^2+\Gamma^2)^2+(2\Gamma\omega)^2}\\ + \frac{\Gamma\omega(\omega^2+\Gamma^2)}{(\Gamma\omega)^2+(3\lambda_{\mathrm{R}}^2)^2}\left[\arctan\frac{\omega+2\sqrt{2}\lambda_{\mathrm{R}}}{\Gamma}\right.\\ + \arctan\frac{\omega-2\sqrt{2}\lambda_{\mathrm{R}}}{\Gamma} + \arctan\frac{\omega+2\lambda_{\mathrm{R}}}{\Gamma}\\ \left.+ \arctan\frac{\omega-2\lambda_{\mathrm{R}}}{\Gamma}\right] - \frac{2\Gamma\omega}{3\lambda_{\mathrm{R}}^2}\left[\arctan\frac{\omega+2\sqrt{2}\lambda_{\mathrm{R}}}{\Gamma}\right.\\ + \arctan\frac{\omega-2\sqrt{2}\lambda_{\mathrm{R}}}{\Gamma} - \arctan\frac{\omega+2\lambda_{\mathrm{R}}}{\Gamma}\\ \left.- \arctan\frac{\omega-2\lambda_{\mathrm{R}}}{\Gamma}\right].\end{aligned} \tag{18}$$

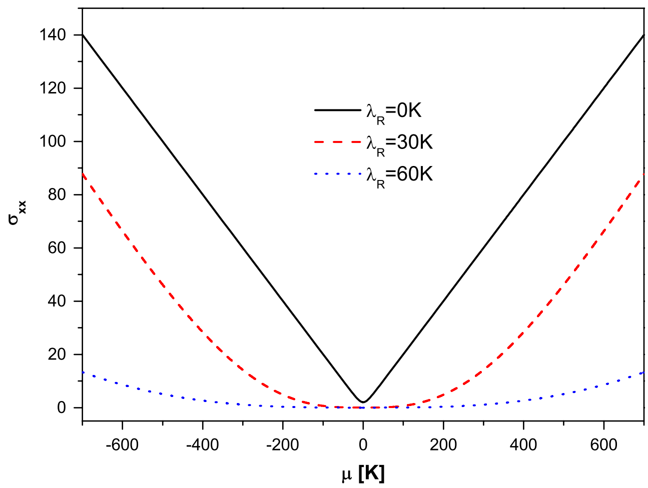

Figure 1. The longitudinal conductivity σ_{xx} measured in $2e^2/h$ units as a function of the chemical potential μ for different values of λ_{R}. We choose $B = 0$ T, $T = 3$ K, $\Gamma = 5$ K and $\lambda_{\mathrm{SO}} = 0.001$ K.

When $T \to 0$, $|\mu| \gg \lambda_{\mathrm{R}}, \Gamma$, the longitudinal conductivity can be further expressed as

$$\sigma_{xx} = \frac{e^2}{2\pi}\frac{|\mu|/\Gamma}{1+(3\lambda_{\mathrm{R}}^2/\Gamma\mu)^2}. \tag{19}$$

When λ_{R} equals zero, equation (19) is consistent with the result in [26].

4. Results and discussion

The numerical results for the electrical conductivity presented below are calculated according to equations (15) and (16). In view of an experimental realization it is advantageous to change units such that $T \to k_{\mathrm{B}}T, eB \to \hbar eBv_{\mathrm{F}}^2$. We focus mainly on Rashba SOI effects on transport; the intrinsic SOI is very small. The strength of the Rashba SOI is tunable by a perpendicular electric field generated by the gate voltage. In this context we mention recent experiments performed on graphene deposited on top of an Ni(111) substrate. A large effective electric field is found due to the formation of a charge density gradient in the interface layer which results in the observed large Rashba splitting in graphene [27]. Figures 1 and 2 show the dependence of the longitudinal conductivity on the chemical potential μ for different Rashba spin–orbit parameters λ_{R} at zero or finite field. In the absence of a B field

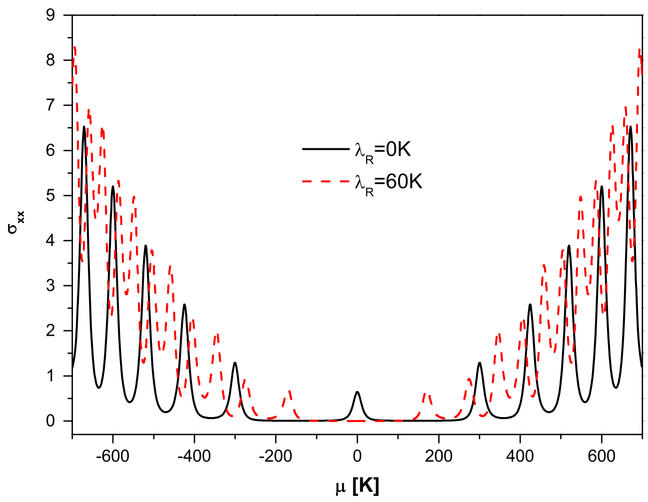

Figure 2. The longitudinal conductivity measured in $2e^2/h$ units as a function of the chemical potential μ for different values of λ_R at $B = 1$ T. The other parameters are the same as in figure 1.

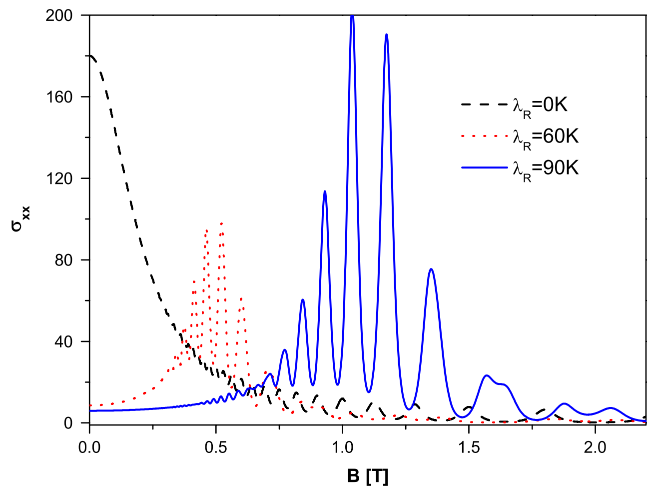

Figure 3. The magnetic field dependence of the longitudinal conductivity measured in $2e^2/h$ units for different values of λ_R at $\mu = -600$ K. The other parameters are the same as in figure 1.

(see figure 1), and if $\lambda_R = 0$, the conductivity is proportional to $|\mu|$ and tends to the known quantum-limited minimal value $4e^2/\pi h$ at a zero chemical potential [26]. For $\lambda_R \neq 0$, there exists a threshold chemical potential μ_c that increases with increasing λ_R. When the chemical potential is smaller than μ_c the longitudinal conductivity becomes almost independent of μ. For $\mu > \mu_c$ the σ_{xx}–μ curves recover the linear relation. This tendency is also readable from equation (19). When a field is applied (cf. figure 2) we observe SdH oscillations of the conductivity due to the Landau level crossing of the Fermi level [28]. If $\lambda_R \neq 0$ each oscillation peak is split into two peaks and the split peaks are shifted by λ_R (figure 2). This behavior is due to the spin–orbit splitting of the Landau levels.

The longitudinal conductivity as a function of the magnetic field B for the different λ_R is shown in figure 3. For $\lambda_R = 0$, the longitudinal conductivity decreases and the intervals between the neighboring SdH oscillation peaks become large with increasing B. The reason is that, in the presence of the magnetic field, only transitions between neighboring Landau levels contribute to electrical conductivity, while a further increase of the magnetic field leads to increasing of the distance between neighboring Landau levels, thus suppressing transitions between them. When B is large enough, the conductivity becomes independent of B since the lowest Landau level is filled which is always below the Fermi level. These observations are quite consistent with the previous studies [26, 28]. In particular, we note that, when the Rashba SOI is present, the longitudinal conductivity exhibits an interesting feature: the SdH oscillations are strongly enhanced at certain positions and damped at others. Such an SdH beating pattern has been observed in a two-dimensional electron gas [29]. The explanation for this phenomenon is as follows: due to the presence of the Rashba SOI two sets of Landau levels emerge with the associated SdH oscillations. The beating pattern occurs due to a superposition of these oscillations. From figure 3 it is clear the enhanced positions and the amplitudes of the SdH oscillations are tunable by varying the strength of the Rashba SOI (e.g. by applying a bias voltage) which results in a shift of one set of the Landau levels (because λ_R is then changed).

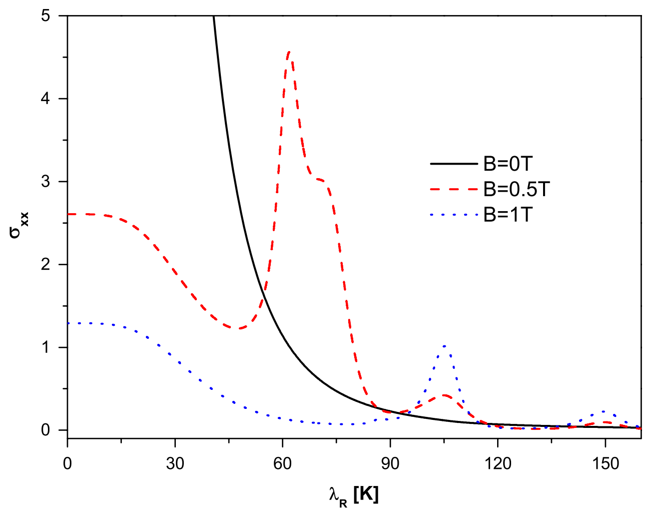

Figure 4. The longitudinal conductivity measured in $2e^2/h$ units as a function of λ_R for different magnetic fields B at $\mu = -300$ K. The other parameters are the same as in figure 1.

Figure 4 shows the longitudinal conductivity versus the Rashba SOI parameter λ_R for different magnetic fields B. For finite B the longitudinal conductivity oscillates as a function of λ_R. This is because the Rashba SOI leads to a shift of the Landau levels. The longitudinal conductivity shows a maximum each time a Landau level passes through the Fermi level of the system, and a minimum when the Fermi level is situated between two Landau levels. Figure 5 shows the temperature dependence of the longitudinal conductivity for different Rashba SOI parameters λ_R at half-filling and away from it. For $\mu = B = 0$, the longitudinal conductivity increases when the temperature grows. However, if the magnetic field and the chemical potential are nonzero, the longitudinal conductivity decreases

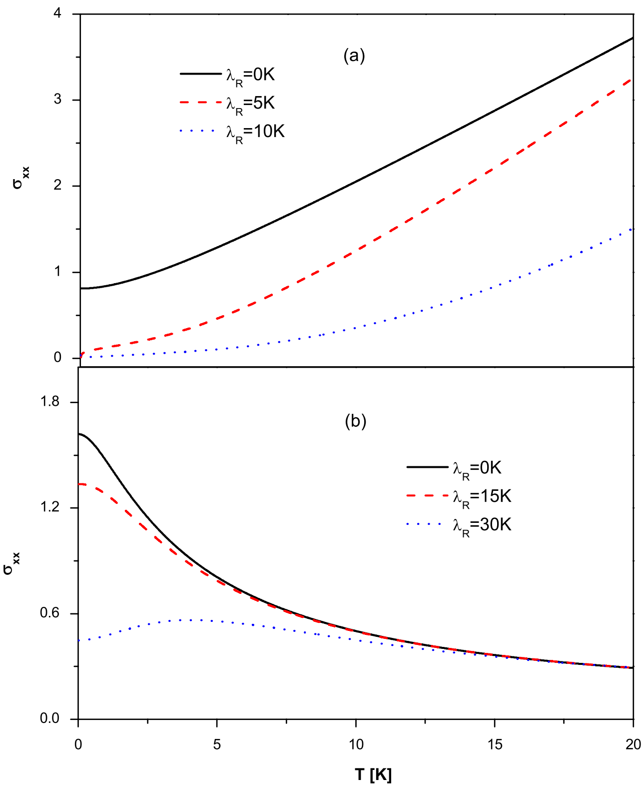

Figure 5. The longitudinal conductivity measured in $2e^2/h$ units as a function of the temperature T at the different λ_R. (a) For $\mu = 0$ K and $B = 0$ T; (b) for $\mu = 300$ K, $B = 1$ T. The other parameters are the same as in figure 1.

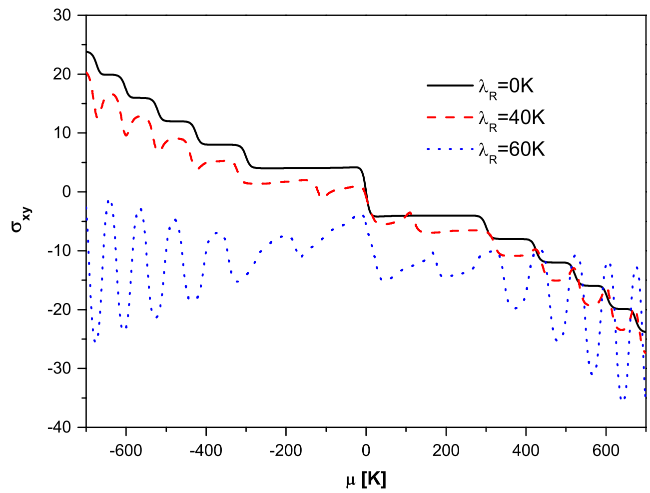

Figure 6. The Hall conductivity measured in $2e^2/h$ units as a function of the chemical potential μ for the different values of λ_R at $B = 1$ T. The other parameters are the same as in figure 1.

with increasing temperature. These observations are in line with previous studies [30]. It is noteworthy that, when the Rashba SOI is taken into account, the longitudinal conductivity becomes non-monotonic (figure 5(b)). After an initial increase it decreases as the temperature increases. It is clear that the Rashba SOI lifts the level degeneracy, providing thus additional channels for the electrons and leading to an enhancement in the longitudinal conductivity. With increasing temperatures, after these channels are occupied completely by thermally excited electrons, the longitudinal conductivity decreases due to thermal fluctuation.

Figure 6 shows Hall conductivity as a function of the chemical potential μ for different λ_R. When $\lambda_R = 0$, the Hall conductivity has a step-like structure as a function of μ which signifies the quantum Hall effect. While the Rashba SOI becomes operative the Hall steps narrow and the step near $\mu = 0$ is split into two steps. The Hall conductivity displays peaks instead of a plateau at larger λ_R. There is no Hall plateau in the cases of sufficiently strong Rashba SOI. This result indicates that the Rashba SOI tends to suppress the quantum Hall effect in graphene.

5. Summary

We investigated the effect of the SOIs on the transport of Dirac fermions in graphene. Our approach is based on the Schwinger proper-time method, the decomposition over Landau level poles and the Kubo formula. We provided analytical expressions for both the longitudinal and the Hall conductivities. We find, when the Rashba SOI is present, the longitudinal conductivity versus the chemical potential deviates from the linear relationship at zero magnetic field. For finite magnetic fields, SdH oscillations in the longitudinal conductivity are observed; each SdH oscillation peak is split into two peaks due to the Rashba SOI. The oscillation in the longitudinal conductivity as a function of the magnetic field shows a beating pattern when the Rashba SOI is turned on. The temperature dependence of the longitudinal conductivity is non-monotonic at finite fields and away from half-filling. Our results also indicate a suppression of the quantum Hall effect in graphene by the Rashba SOI.

Acknowledgments

The work of KHD was supported by the Natural Science Foundation of Hunan Province, China (grant no. 08JJ4002) and by the Education Department of Hunan Province, China. JB was supported by the cluster of excellence 'Nanostructured Materials' of the state Saxony-Anhalt.

Appendix A

Here we provide all the details sufficient to reproduce the complete results discussed above. The Green's function $G_i^{(R,A)}(\omega, \mathbf{k})$ is given by

$$\begin{aligned}G_1^{(R,A)}(\omega, \mathbf{k}) &= (H_s + \gamma^0(\omega \pm \mathrm{i}\Gamma))[A_1^{(R,A)}(\omega) \\ &- \mathrm{i}\gamma^1\gamma^2 \mathrm{sgn}(eB) A_2^{(R,A)}(\omega)] \\ &- \mathrm{i}(\gamma^1 k_2 - \gamma^2 k_1)\mathrm{sgn}(eB)[B_1^{(R,A)}(\omega) - A_2^{(R,A)}(\omega)] \\ &+ (\gamma^1 k_1 + \gamma^2 k_2)[B_2^{(R,A)}(\omega) - A_1^{(R,A)}(\omega)],\end{aligned} \tag{A.1}$$

$$\begin{aligned}G_2^{(R,A)}(\omega, \mathbf{k}) &= (H_s + \gamma^0(\omega \pm \mathrm{i}\Gamma))\gamma^0 s_z[A_3^{(R,A)}(\omega) \\ &- \mathrm{i}\gamma^1\gamma^2 \mathrm{sgn}(eB) A_4^{(R,A)}(\omega)] \\ &- \mathrm{i}(\gamma^1 k_2 - \gamma^2 k_1)\gamma^0 s_z \mathrm{sgn}(eB)[B_3^{(R,A)}(\omega) - A_4^{(R,A)}(\omega)] \\ &+ (\gamma^1 k_1 + \gamma^2 k_2)\gamma^0 s_z[B_4^{(R,A)}(\omega) - A_3^{(R,A)}(\omega)],\end{aligned} \tag{A.2}$$

$$
\begin{aligned}
G_3^{(R,A)}(\omega,\mathbf{k}) &= -\frac{12\lambda_{\mathrm{SO}}\lambda_{\mathrm{R}}}{\xi}(H_s+\gamma^0(\omega\pm \mathrm{i}\Gamma))\\
&\times(\mathrm{i}\gamma^1 s_y-\gamma^0\gamma^1 s_x)(I_4^{(R,A)}(\omega)+\gamma^0 s_z I_8^{(R,A)}(\omega))\\
&+\frac{12\lambda_{\mathrm{SO}}\lambda_{\mathrm{R}}}{\xi}(\gamma^1k_1+\gamma^2k^2)(I_4^{(R,A)}(\omega)\\
&+\mathrm{i}\gamma^1\gamma^2\,\mathrm{sgn}(eB)J_4^{(R,A)}(\omega))(\mathrm{i}\gamma^1 s_y-\gamma^0\gamma^1 s_x)\\
&+\frac{12\lambda_{\mathrm{SO}}\lambda_{\mathrm{R}}}{\xi}(\gamma^1k_1+\gamma^2k^2)(I_8^{(R,A)}(\omega)\\
&+\mathrm{i}\gamma^1\gamma^2\,\mathrm{sgn}(eB)J_8^{(R,A)}(\omega))(\mathrm{i}\gamma^1 s_y-\gamma^0\gamma^1 s_x)\gamma^0 s_z,
\end{aligned}\tag{A.3}
$$

where $A_1 = I_1+I_3+I_5-I_7$, $A_2 = I_2+\frac{1}{\chi}I_4+I_6-\frac{1}{\chi}I_8$, $A_3 = I_1-I_3+I_5+I_7$, $A_4 = I_2-\frac{1}{\chi}I_4+I_6+\frac{1}{\chi}I_8$, $B_1 = I_2+J_2+I_6-J_5$, $B_2 = J_1+\frac{1}{\chi}J_3+J_4-\frac{1}{\chi}J_6$, $B_3 = I_2-J_2+I_6+J_5$, $B_4 = J_1-\frac{1}{\chi}J_3+J_4+\frac{1}{\chi}J_6$ and

$$
\begin{aligned}
I_1^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\\
&\times\sum_{n=0}^{\infty}(-1)^n\left[\frac{L_n(2\alpha)-L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta+2n|eB|}\right.\\
&\left.+\frac{L_n(2\alpha)-L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+2n|eB|}\right],
\end{aligned}\tag{A.4}
$$

$$
\begin{aligned}
I_2^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\\
&\times\sum_{n=0}^{\infty}(-1)^n\left[\frac{L_n(2\alpha)+L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta+2n|eB|}\right.\\
&\left.+\frac{L_n(2\alpha)+L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+2n|eB|}\right],
\end{aligned}\tag{A.5}
$$

$$
\begin{aligned}
I_3^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\sum_{n=0}^{\infty}(-1)^n\\
&\times\left[\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta-\xi+(2n+1)|eB|}\right.\\
&+\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+\xi+(2n+1)|eB|}\\
&+\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta+\xi+(2n+1)|eB|}\\
&\left.+\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta-\xi+(2n+1)|eB|}\right],
\end{aligned}\tag{A.6}
$$

$$
\begin{aligned}
I_4^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\sum_{n=0}^{\infty}(-1)^n\\
&\times\left[\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta-\xi+(2n+1)|eB|}\right.\\
&-\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+\xi+(2n+1)|eB|}\\
&-\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta+\xi+(2n+1)|eB|}\\
&\left.+\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta-\xi+(2n+1)|eB|}\right],
\end{aligned}\tag{A.7}
$$

$$
\begin{aligned}
I_5^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\\
&\times\sum_{n=0}^{\infty}(-1)^n\left[\frac{L_n(2\alpha)-L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta+2n|eB|}\right.\\
&\left.-\frac{L_n(2\alpha)-L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+2n|eB|}\right],
\end{aligned}\tag{A.8}
$$

$$
\begin{aligned}
I_6^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\\
&\times\sum_{n=0}^{\infty}(-1)^n\left[\frac{L_n(2\alpha)+L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta+2n|eB|}\right.\\
&\left.-\frac{L_n(2\alpha)+L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+2n|eB|}\right],
\end{aligned}\tag{A.9}
$$

$$
\begin{aligned}
I_7^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\sum_{n=0}^{\infty}(-1)^n\\
&\times\left[\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta-\xi+(2n+1)|eB|}\right.\\
&-\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+\xi+(2n+1)|eB|}\\
&+\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta+\xi+(2n+1)|eB|}\\
&\left.-\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta-\xi+(2n+1)|eB|}\right],
\end{aligned}\tag{A.10}
$$

$$
\begin{aligned}
I_8^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\sum_{n=0}^{\infty}(-1)^n\\
&\times\left[\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta-\xi+(2n+1)|eB|}\right.\\
&+\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+\xi+(2n+1)|eB|}\\
&-\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta+\xi+(2n+1)|eB|}\\
&\left.-\frac{L_n(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta-\xi+(2n+1)|eB|}\right],
\end{aligned}\tag{A.11}
$$

$$
\begin{aligned}
J_1^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\\
&\times\sum_{n=0}^{\infty}(-1)^n\left[\frac{4L_{n-1}^1(2\alpha)+L_n(2\alpha)-L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta+2n|eB|}\right.\\
&\left.+\frac{4L_{n-1}^1(2\alpha)+L_n(2\alpha)-L_{n-1}(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+2n|eB|}\right],
\end{aligned}\tag{A.12}
$$

$$
\begin{aligned}
J_2^{(R,A)}(\omega) &= \tfrac{1}{2}\mathrm{e}^{-\alpha}\sum_{n=0}^{\infty}(-1)^n\\
&\times\left[\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2-\zeta-\xi+(2n+1)|eB|}\right.\\
&+\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm \mathrm{i}\Gamma)^2+10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2+\zeta+\xi+(2n+1)|eB|}
\end{aligned}
$$

$$+\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2-\zeta+\xi+(2n+1)|eB|}$$
$$\left.+\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2+\zeta-\xi+(2n+1)|eB|}\right], \tag{A.13}$$

$$J_3^{(R,A)}(\omega)=\tfrac{1}{2}e^{-\alpha}\sum_{n=0}^{\infty}(-1)^n$$
$$\times\left[\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2-\zeta-\xi+(2n+1)|eB|}\right.$$
$$-\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2+\zeta+\xi+(2n+1)|eB|}$$
$$-\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2-\zeta+\xi+(2n+1)|eB|}$$
$$\left.+\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2+\zeta-\xi+(2n+1)|eB|}\right], \tag{A.14}$$

$$J_4^{(R,A)}(\omega)=\tfrac{1}{2}e^{-\alpha}\sum_{n=0}^{\infty}(-1)^n$$
$$\times\left[\frac{4L_{n-1}^1(2\alpha)+L_n(2\alpha)-L_{n-1}(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2-\zeta+2n|eB|}\right.$$
$$\left.-\frac{4L_{n-1}^1(2\alpha)+L_n(2\alpha)-L_{n-1}(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2+\zeta+2n|eB|}\right], \tag{A.15}$$

$$J_5^{(R,A)}(\omega)=\tfrac{1}{2}e^{-\alpha}\sum_{n=0}^{\infty}(-1)^n$$
$$\times\left[\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2-\zeta-\xi+(2n+1)|eB|}\right.$$
$$-\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2+\zeta+\xi+(2n+1)|eB|}$$
$$+\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2-\zeta+\xi+(2n+1)|eB|}$$
$$\left.-\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2+\zeta-\xi+(2n+1)|eB|}\right], \tag{A.16}$$

$$J_6^{(R,A)}(\omega)=\tfrac{1}{2}e^{-\alpha}\sum_{n=0}^{\infty}(-1)^n$$
$$\times\left[\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2-\zeta-\xi+(2n+1)|eB|}\right.$$
$$-\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2+\zeta+\xi+(2n+1)|eB|}$$
$$+\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2-\zeta+\xi+(2n+1)|eB|}$$
$$\left.-\frac{L_n^1(2\alpha)+L_{n-1}^1(2\alpha)}{(\omega\pm i\Gamma)^2+10\lambda_{SO}^2+2\lambda_R^2+\zeta-\xi+(2n+1)|eB|}\right], \tag{A.17}$$

with $L_n^i(z)$ being the generalized Laguerre polynomials, $\alpha=-\frac{\mathbf{k}^2}{|eB|}$, and $\chi=\frac{\xi}{|eB|}$.

Appendix B

Substituting equation (11) in (14), the traces in equation (14) are evaluated after lengthy and involved calculations:

$$\begin{aligned}
&\mathrm{tr}\{\gamma^i G^{(R,A)}(\omega',\mathbf{k})\gamma^j G^{(R,A)}(\omega,\mathbf{k})\}\\
&=\delta_{ij}(\omega'\pm i\Gamma)(\omega\pm i\Gamma)[A_1^{(R,A)}(\omega')A_1^{(R,A)}(\omega)\\
&\quad-A_2^R(\omega')A_2^A(\omega)+A_4^{(R,A)}(\omega')A_4^{(R,A)}(\omega)\\
&\quad-A_3^{(R,A)}(\omega')A_3^{(R,A)}(\omega)]+i\epsilon_{ij}\,\mathrm{sgn}(eB)(\omega'\pm i\Gamma)\\
&\quad\times(\omega\pm i\Gamma)[A_2^{(R,A)}(\omega')A_1^{(R,A)}(\omega)-A_1^{(R,A)}(\omega')A_2^{(R,A)}(\omega)\\
&\quad+A_3^{(R,A)}(\omega')A_4^{(R,A)}(\omega)-A_4^{(R,A)}(\omega')A_3^{(R,A)}(\omega)]\\
&\quad+(2k_ik_j-\delta_{ij}\mathbf{k}^2)[(B_1^{(R,A)}(\omega')-A_2^{(R,A)}(\omega'))(B_1^{(R,A)}(\omega)\\
&\quad-A_2^{(R,A)}(\omega))+(B_2^{(R,A)}(\omega')-A_1^{(R,A)}(\omega'))(B_2^{(R,A)}(\omega)\\
&\quad-A_1^{(R,A)}(\omega))+(B_3^{(R,A)}(\omega')-A_4^{(R,A)}(\omega'))(B_3^{(R,A)}(\omega)\\
&\quad-A_4^{(R,A)}(\omega))+(B_4^{(R,A)}(\omega')-A_3^{(R,A)}(\omega'))(B_4^{(R,A)}(\omega)\\
&\quad-A_3^{(R,A)}(\omega))]-i\,\mathrm{sgn}(eB)[\delta_{ij}(-1)^{i-1}2k_1k_2\\
&\quad+\epsilon_{ij}(k_j^2-k_i^2)][(B_1^{(R,A)}(\omega')-A_2^{(R,A)}(\omega'))(B_2^{(R,A)}(\omega)\\
&\quad-A_1^{(R,A)}(\omega))+(B_3^{(R,A)}(\omega')-A_4^{(R,A)}(\omega'))(B_4^{(R,A)}(\omega)\\
&\quad-A_3^{(R,A)}(\omega))+(B_2^{(R,A)}(\omega')-A_1^{(R,A)}(\omega'))(B_1^{(R,A)}(\omega)\\
&\quad-A_2^{(R,A)}(\omega))+(B_4^{(R,A)}(\omega')-A_3^{(R,A)}(\omega'))(B_3^{(R,A)}(\omega)\\
&\quad-A_4^{(R,A)}(\omega))]+\delta_{ij}2\left(\frac{12\lambda_{SO}\lambda_R}{\xi}\right)^2(I_4^{(R,A)}(\omega')I_4^{(R,A)}(\omega)\\
&\quad-J_4^{(R,A)}(\omega')J_4-I_8^{(R,A)}(\omega')I_8^{(R,A)}(\omega)\\
&\quad+J_8^{(R,A)}(\omega')J_8^{(R,A)}(\omega))\mathbf{k}^2\\
&\quad-i\epsilon_{ij}\mathrm{sgn}(eB)2\left(\frac{12\lambda_{SO}\lambda_R}{\xi}\right)^2[I_4^{(R,A)}(\omega')J_4^{(R,A)}(\omega)\\
&\quad-J_4^{(R,A)}(\omega')I_4^{(R,A)}(\omega)-I_8^{(R,A)}(\omega')J_8^{(R,A)}(\omega)\\
&\quad+J_8^{(R,A)}(\omega')I_8^{(R,A)}(\omega)]\mathbf{k}^2,
\end{aligned} \tag{B.1}$$

where ϵ_{ij} is an antisymmetric tensor ($\epsilon_{12}=1$). Integrating over momenta in equation (14), we obtain the longitudinal conductivity:

$$\sigma_{xx}=\sigma_{xx}(\Omega\to 0)=\frac{2e^2v_F^2|eB|}{\pi^2}\mathrm{Re}\int_{-\infty}^{\infty}d\omega\frac{1}{4T\cosh^2\frac{\beta(\omega-\mu)}{2}}$$
$$\times\left\{X_L+\frac{1}{2|eB|}Y_L-\left(\frac{6\lambda_{SO}\lambda_R}{\xi}\right)^2Z_L\right\}, \tag{B.2}$$

where

$$X_L=\frac{1}{(\omega+i\Gamma)^2-(10\lambda_{SO}^2+2\lambda_R^2)-\zeta-\xi+|eB|}$$
$$\times\left[\frac{2(\omega^2+\Gamma^2)}{i4\Gamma\omega-2\zeta-\xi+|eB|-2|eB|}\right.$$
$$\left.-\frac{2(\omega+i\Gamma)^2}{-2\zeta-\xi+|eB|-2|eB|}\right]$$
$$+\frac{(1-\frac{1}{\chi})}{(\omega+i\Gamma)^2-(10\lambda_{SO}^2+2\lambda_R^2)-\zeta+\xi+|eB|}$$
$$\times\left[\frac{(\omega^2+\Gamma^2)}{i4\Gamma\omega-2\zeta+\xi+|eB|-2|eB|}\right.$$

$$
\left.- \frac{(\omega + i\Gamma)^2}{-2\zeta + \xi + |eB| - 2|eB|}\right]
- \frac{1}{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta}
\times \left[\frac{(1 - \frac{1}{\chi})(\omega^2 + \Gamma^2)}{-i4\Gamma\omega - 2\zeta - \xi - |eB| + 2|eB|}
- \frac{(1 - \frac{1}{\chi})(\omega + i\Gamma)^2}{-2\zeta - \xi - |eB| + 2|eB|}
+ \frac{2(\omega^2 + \Gamma^2)}{-i4\Gamma\omega - 2\zeta + \xi - |eB| + 2|eB|}
- \frac{2(\omega + i\Gamma)^2}{-2\zeta + \xi - |eB| + 2|eB|} \right], \tag{B.3}
$$

$$
\begin{aligned}
Y_L = {} & w_1\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta}{2|eB|}\right) \\
& + w_2\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta - \xi + |eB|}{2|eB|}\right) \\
& + w_3\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta + \xi + |eB|}{2|eB|}\right) \\
& + w_4\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta + \xi - |eB|}{2|eB|}\right) \\
& + w_5\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta - \xi - |eB|}{2|eB|}\right),
\end{aligned} \tag{B.4}
$$

$$
\begin{aligned}
Z_L = {} & z_1\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta + \xi - |eB|}{2|eB|}\right) \\
& - z_2\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta - \xi + |eB|}{2|eB|}\right) \\
& + z_3\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta + \xi + |eB|}{2|eB|}\right) \\
& - z_4\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta - \xi - |eB|}{2|eB|}\right) \\
& + z_5\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta - \xi - |eB|}{2|eB|}\right) \\
& - z_6\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta + \xi + |eB|}{2|eB|}\right) \\
& + z_7\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta - \xi + |eB|}{2|eB|}\right) \\
& - z_8\psi\left(-\frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta + \xi - |eB|}{2|eB|}\right),
\end{aligned} \tag{B.5}
$$

with $\psi(z)$ being the Digamma function ($\psi(z) = \partial_z \log\Gamma(z)$ and $\Gamma(z)$ is the Gamma function) and

$$
\begin{aligned}
w_1 = {} & \frac{2(\omega^2 + \Gamma^2)}{-i4\Gamma\omega - 2\zeta - \xi + |eB| - 2|eB|}
- \frac{2(\omega + i\Gamma)^2}{-2\zeta - \xi + |eB| - 2|eB|}
+ \frac{(1 - \frac{1}{\chi})(\omega^2 + \Gamma^2)}{-i4\Gamma\omega - 2\zeta + \xi + |eB| - 2|eB|} \\
& - \frac{(1 - \frac{1}{\chi})(\omega + i\Gamma)^2}{-2\zeta + \xi + |eB| - 2|eB|}
+ \frac{(1 - \frac{1}{\chi})(\omega^2 + \Gamma^2)}{-i4\Gamma\omega - 2\zeta - \xi - |eB| + 2|eB|}
- \frac{(1 - \frac{1}{\chi})(\omega + i\Gamma)^2}{-2\zeta - \xi - |eB| + 2|eB|} \\
& + \frac{2(\omega^2 + \Gamma^2)}{-i4\Gamma\omega - 2\zeta + \xi - |eB| + 2|eB|}
- \frac{2(\omega + i\Gamma)^2}{-2\zeta + \xi - |eB| + 2|eB|},
\end{aligned}
$$

$$
w_2 = \frac{2(\omega^2 + \Gamma^2)}{-i4\Gamma\omega + 2\zeta + \xi - |eB| + 2|eB|}
- \frac{2(\omega + i\Gamma)^2}{2\zeta + \xi - |eB| + 2|eB|},
$$

$$
w_3 = \frac{(1 - \frac{1}{\chi})(\omega^2 + \Gamma^2)}{-i4\Gamma\omega + 2\zeta - \xi - |eB| + 2|eB|}
- \frac{(1 - \frac{1}{\chi})(\omega + i\Gamma)^2}{2\zeta - \xi - |eB| + 2|eB|},
$$

$$
w_4 = \frac{2(\omega^2 + \Gamma^2)}{-i4\Gamma\omega + 2\zeta - \xi + |eB| - 2|eB|}
- \frac{2(\omega + i\Gamma)^2}{2\zeta - \xi + |eB| - 2|eB|},
$$

$$
w_5 = \frac{(1 - \frac{1}{\chi})(\omega^2 + \Gamma^2)}{-i4\Gamma\omega + 2\zeta + \xi + |eB| - 2|eB|}
- \frac{(1 - \frac{1}{\chi})(\omega + i\Gamma)^2}{2\zeta + \xi + |eB| - 2|eB|},
$$

$$
\begin{aligned}
z_1 = {} & \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta + \xi - |eB|}{-i4\Gamma\omega - 2\zeta - 2\xi + 2|eB|}
- \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta + \xi - |eB|}{-i4\Gamma\omega - 2\zeta + 2|eB|} \\
& - \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta + \xi - |eB|}{-2\zeta - 2\xi + 2|eB|}
+ \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta + \xi - |eB|}{-2\zeta + 2|eB|},
\end{aligned}
$$

$$
\begin{aligned}
z_2 = {} & \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta - \xi + |eB|}{i4\Gamma\omega - 2\zeta - 2\xi + 2|eB|}
- \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta - \xi + |eB|}{i4\Gamma\omega - 2\zeta + 2|eB|} \\
& - \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta - \xi + |eB|}{-2\zeta - 2\xi + 2|eB|}
+ \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta - \xi + |eB|}{-2\zeta + 2|eB|},
\end{aligned}
$$

$$
z_3 = \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta + \xi + |eB|}{i4\Gamma\omega - 2\zeta + 2|eB|}
- \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta + \xi + |eB|}{i4\Gamma\omega - 2\zeta + 2\xi + 2|eB|}
$$

$$
\begin{aligned}
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{-2\zeta+2|eB|}\\
&+\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{-2\zeta+2\xi+2|eB|},\\
z_4=&\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta+2\xi+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{-2\zeta+2|eB|}\\
&+\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{-2\zeta+2\xi+2|eB|},\\
z_5=&\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta+2\xi+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2\zeta+2\xi+2|eB|}\\
&+\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2\zeta+2|eB|},\\
z_6=&\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{\mathrm{i}4\Gamma\omega+2\zeta+2\xi+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{\mathrm{i}4\Gamma\omega+2\zeta+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{2\zeta+2\xi+2|eB|}\\
&+\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{2\zeta+2|eB|},\\
z_7=&\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi+|eB|}{\mathrm{i}4\Gamma\omega+2\zeta+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi+|eB|}{\mathrm{i}4\Gamma\omega+2\zeta-2\xi+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi+|eB|}{2\zeta+2|eB|}\\
&+\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi+|eB|}{2\zeta-2\xi+2|eB|},\\
z_8=&\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta-2\xi+2|eB|}\\
&-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2\zeta+2|eB|}\\
&+\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2\zeta-2\xi+2|eB|}.
\end{aligned}
$$

The Hall conductivity is given by

$$
\begin{aligned}
\sigma_{xy}=\sigma_{xy}(\Omega\to 0)=&-\frac{2e^2v_{\mathrm{F}}^2|eB|\mathrm{sgn}(eB)}{\pi^2}\\
&\times\ \mathrm{Im}\int_{-\infty}^{\infty}\mathrm{d}\omega\frac{1}{4T\cosh^2\frac{\beta(\omega-\mu)}{2}}\\
&\times\left[X_H+\frac{1}{2|eB|}Y_H+\left(\frac{6\lambda_{\mathrm{SO}}\lambda_{\mathrm{R}}}{\xi}\right)^2 Z_H\right],
\end{aligned}
\tag{B.6}
$$

here

$$
\begin{aligned}
X_H=a_1/2-a_1',\qquad Y_H=b_1/2-(b_1'+b_2'),\\
Z_H=c_1/2-c_1',
\end{aligned}
\tag{B.7}
$$

where

$$
\begin{aligned}
a_1=&\frac{2(1-\frac{1}{\chi})}{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}\\
&\times\frac{\omega^2+\Gamma^2}{-\mathrm{i}4\Gamma\omega+2\zeta-\xi-|eB|+2|eB|}\\
&+\frac{2(1+\frac{1}{\chi})}{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}\\
&\times\frac{\omega^2+\Gamma^2}{-\mathrm{i}4\Gamma\omega+2\zeta+\xi-|eB|+2|eB|}\\
&+\frac{\omega^2+\Gamma^2}{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta}\\
&\times\left[\frac{2(1-\frac{1}{\chi})}{-\mathrm{i}4\Gamma\omega-2\zeta-\xi-|eB|+2|eB|}\right.\\
&\left.+\frac{2(1+\frac{1}{\chi})}{-\mathrm{i}4\Gamma\omega-2\zeta+\xi-|eB|+2|eB|}\right],
\end{aligned}
\tag{B.8}
$$

$$
\begin{aligned}
b_1=&\mu_1\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta}{2|eB|}\right)\\
&-\mu_2\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{2|eB|}\right)\\
&-\mu_3\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}{2|eB|}\right)\\
&+\mu_4\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2|eB|}\right)\\
&+\mu_5\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2|eB|}\right),
\end{aligned}
\tag{B.9}
$$

$$
\begin{aligned}
c_1=&\nu_1\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi-|eB|}{2|eB|}\right)\\
&-\nu_2\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{2|eB|}\right)\\
&+\nu_3\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2|eB|}\right)\\
&-\nu_4\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2|eB|}\right)\\
&-\nu_5\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{2|eB|}\right)
\end{aligned}
$$

$$+\nu_6\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi+|eB|}{2|eB|}\right)$$
$$-\nu_7\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}{2|eB|}\right)$$
$$+\nu_8\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{2|eB|}\right), \quad \text{(B.10)}$$

$$a_1' = \alpha_0+\alpha_1\ln[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|]$$
$$+\alpha_2\ln[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|]$$
$$+\alpha_3\ln[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta], \quad \text{(B.11)}$$

$$b_1' = \beta_1\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta}{2|eB|}\right)$$
$$+\beta_2\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta}{2|eB|}\right)$$
$$+\beta_3\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta}{2|eB|}\right), \quad \text{(B.12)}$$

$$b_2' = \beta_4\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}{2|eB|}\right)$$
$$+\beta_5\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}{2|eB|}\right)$$
$$-\beta_6\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}{2|eB|}\right)$$
$$+\beta_7\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{2|eB|}\right)$$
$$+\beta_8\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{2|eB|}\right)$$
$$-\beta_9\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{2|eB|}\right)$$
$$-\beta_{10}\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2|eB|}\right)$$
$$-\beta_{11}\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2|eB|}\right)$$
$$+\beta_{12}\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2|eB|}\right)$$
$$-\beta_{13}\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2|eB|}\right)$$
$$-\beta_{14}\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2|eB|}\right)$$
$$+\beta 15\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2|eB|}\right), \quad \text{(B.13)}$$

$$c_1' = \gamma_1\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_4\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_7\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_{10}\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_{13}\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_{16}\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_{19}\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_{22}\ln\Gamma\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_2\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_6\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_9\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_{12}\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_{15}\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_{18}\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_{21}\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_{24}\psi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_3\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_5\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_8\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_{11}\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2|eB|}\right)$$
$$+\gamma_{14}\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_{17}\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_{23}\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}{2|eB|}\right)$$
$$+\gamma_{20}\phi\left(-\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}{2|eB|}\right) \quad \text{(B.14)}$$

with $\phi(z)=\int_0^z \ln\Gamma(x)\,\mathrm{d}x$:

$$\mu_1=\frac{2(1+\frac{1}{\chi})(\omega^2+\Gamma^2)}{-\mathrm{i}4\Gamma\omega-2\zeta-\xi+|eB|-2|eB|}-\frac{2(1+\frac{1}{\chi})(\omega^2+\Gamma^2)}{-\mathrm{i}4\Gamma\omega-2\zeta+\xi-|eB|+2|eB|}+\frac{2(1-\frac{1}{\chi})(\omega^2+\Gamma^2)}{-\mathrm{i}4\Gamma\omega-2\zeta+\xi+|eB|-2|eB|}-\frac{2(1-\frac{1}{\chi})(\omega^2+\Gamma^2)}{-\mathrm{i}4\Gamma\omega-2\zeta-\xi-|eB|+2|eB|},\quad \text{(B.15)}$$

$$\mu_2=\frac{2(1-\frac{1}{\chi})(\omega^2+\Gamma^2)}{-\mathrm{i}4\Gamma\omega+2\zeta-\xi-|eB|+2|eB|},\quad \text{(B.16)}$$

$$\mu_3=\frac{2(1+\frac{1}{\chi})(\omega^2+\Gamma^2)}{-\mathrm{i}4\Gamma\omega+2\zeta+\xi-|eB|+2|eB|},$$

$$\mu_4=\frac{2(1+\frac{1}{\chi})(\omega^2+\Gamma^2)}{-\mathrm{i}4\Gamma\omega+2\zeta-\xi+|eB|-2|eB|},\quad \text{(B.17)}$$

$$\mu_5=\frac{2(1-\frac{1}{\chi})(\omega^2+\Gamma^2)}{-\mathrm{i}4\Gamma\omega+2\zeta+\xi+|eB|-2|eB|},$$

$$\nu_1=2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2+\zeta+\xi-|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta-2\xi+2|eB|}-2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2+\zeta+\xi-|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta+2|eB|},\quad \text{(B.18)}$$

$$\nu_2=2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2+\zeta-\xi-|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta+2|eB|}-2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2+\zeta-\xi-|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta+2\xi+2|eB|},\quad \text{(B.19)}$$

$$\nu_3=2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2-\zeta-\xi-|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta+2\xi+2|eB|}-2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2-\zeta-\xi-|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta+2|eB|},\quad \text{(B.20)}$$

$$\nu_4=2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2-\zeta+\xi-|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta+2|eB|}-2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2-\zeta+\xi-|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta-2\xi+2|eB|},\quad \text{(B.21)}$$

$$\nu_5=2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2+\zeta+\xi+|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta-2\xi-2|eB|}-2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2+\zeta+\xi-|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta-2|eB|},\quad \text{(B.22)}$$

$$\nu_6=2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2+\zeta-\xi+|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta-2|eB|}-2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2+\zeta-\xi+|eB|}{-\mathrm{i}4\Gamma\omega-2\zeta+\xi-2|eB|},\quad \text{(B.23)}$$

$$\nu_7=2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2-\zeta-\xi+|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta+2\xi-2|eB|}-2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2-\zeta-\xi+|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta-2|eB|},\quad \text{(B.24)}$$

$$\nu_8=2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2-\zeta+\xi+|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta-2|eB|}-2\frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}+2\lambda_{\mathrm{R}})^2-\zeta+\xi+|eB|}{-\mathrm{i}4\Gamma\omega+2\zeta-2\xi-2|eB|},\quad \text{(B.25)}$$

$$\begin{aligned}\alpha_0=&\frac{2(1+\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{(2\zeta+\xi-|eB|+2|eB|)^2}+\frac{(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi-|eB|}{2\zeta+\xi-|eB|+2|eB|}\times\frac{\frac{1}{2}(1+\frac{1}{\chi})}{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi+|eB|}\\&+\frac{2(1-\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{(2\zeta-\xi-|eB|+2|eB|)^2}+\frac{(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{2\zeta-\xi-|eB|+2|eB|}\times\frac{\frac{1}{2}(1-\frac{1}{\chi})}{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi+|eB|}\\&+\frac{2(1+\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{(2\zeta-\xi+|eB|-2|eB|)^2}+\frac{(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta}{2\zeta-\xi+|eB|-2|eB|}\times\frac{\frac{1}{2}(1+\frac{1}{\chi})}{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta}\\&+\frac{2(1-\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{(2\zeta+\xi+|eB|-2|eB|)^2}+\frac{(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta}{2\zeta+\xi+|eB|-2|eB|}\times\frac{\frac{1}{2}(1-\frac{1}{\chi})}{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta},\end{aligned}$$

$$\alpha_1=\frac{2(1+\frac{1}{\chi})[(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta+3\xi-5|eB|]}{(2\zeta+\xi-|eB|+2|eB|)^2},$$

$$\alpha_2=\frac{2(1-\frac{1}{\chi})[(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta-3\xi-5|eB|]}{(2\zeta-\xi-|eB|+2|eB|)^2},$$

$$\alpha_3=\frac{2(1+\frac{1}{\chi})[(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-2\zeta-\xi+|eB|-2|eB|]}{(2\zeta-\xi+|eB|-2|eB|)^2}+\frac{2(1-\frac{1}{\chi})[(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-2\zeta+\xi+|eB|-2|eB|]}{(2\zeta+\xi+|eB|-2|eB|)^2},$$

$$\begin{aligned}\beta_1=&-\frac{2(1+\frac{1}{\chi})|eB|[2(\omega+\mathrm{i}\Gamma)^2+2\zeta+\xi-|eB|+2|eB|]}{(2\zeta+\xi-|eB|+2|eB|)^2}-\frac{2(1-\frac{1}{\chi})|eB|[2(\omega+\mathrm{i}\Gamma)^2+2\zeta-\xi-|eB|+2|eB|]}{(2\zeta-\xi-|eB|+2|eB|)^2}\\&+\frac{2(1-\frac{1}{\chi})|eB|[2(\omega+\mathrm{i}\Gamma)^2+2\zeta+\xi+|eB|-2|eB|]}{(2\zeta+\xi+|eB|-2|eB|)^2}+\frac{2(1+\frac{1}{\chi})|eB|[2(\omega+\mathrm{i}\Gamma)^2+2\zeta-\xi+|eB|-2|eB|]}{(2\zeta-\xi+|eB|-2|eB|)^2},\end{aligned}$$

$$\beta_2 = -\frac{(1+\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{2\zeta+\xi-|eB|+2|eB|} - \frac{(1-\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{2\zeta-\xi-|eB|+2|eB|} + \frac{(1-\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{2\zeta+\xi+|eB|-2|eB|} + \frac{(1+\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{2\zeta-\xi+|eB|-2|eB|},$$

$$\beta_3 = -\frac{8|eB|^2(1+\frac{1}{\chi})}{(2\zeta+\xi-|eB|+2|eB|)^2} - \frac{8|eB|^2(1-\frac{1}{\chi})}{(2\zeta-\xi-|eB|+2|eB|)^2} + \frac{8|eB|^2(1-\frac{1}{\chi})}{(2\zeta+\xi+|eB|-2|eB|)^2} + \frac{8|eB|^2(1+\frac{1}{\chi})}{(2\zeta-\xi+|eB|-2|eB|)^2},$$

$$\beta_4 = \frac{2(1+\frac{1}{\chi})|eB|[2(\omega+\mathrm{i}\Gamma)^2-2\zeta-\xi+|eB|-2|eB|]}{(2\zeta+\xi-|eB|+2|eB|)^2},$$

$$\beta_5 = \frac{8(1+\frac{1}{\chi})|eB|^2}{(2\zeta+\xi-|eB|+2|eB|)^2},$$

$$\beta_6 = \frac{(1+\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{2\zeta+\xi-|eB|+2|eB|},$$

$$\beta_7 = \frac{2(1-\frac{1}{\chi})|eB|[2(\omega+\mathrm{i}\Gamma)^2-2\zeta+\xi+|eB|-2|eB|]}{(2\zeta-\xi-|eB|+2|eB|)^2},$$

$$\beta_8 = \frac{8(1-\frac{1}{\chi})|eB|^2}{(2\zeta-\xi-|eB|+2|eB|)^2},$$

$$\beta_9 = \frac{(1-\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{2\zeta-\xi-|eB|+2|eB|},$$

$$\beta_{10} = \frac{2(1-\frac{1}{\chi})|eB|[2(\omega+\mathrm{i}\Gamma)^2-2\zeta-\xi-|eB|+2|eB|]}{(2\zeta+\xi+|eB|-2|eB|)^2},$$

$$\beta_{11} = \frac{8(1-\frac{1}{\chi})|eB|^2}{(2\zeta+\xi+|eB|-2|eB|)^2},$$

$$\beta_{12} = \frac{(1-\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{2\zeta+\xi+|eB|-2|eB|},$$

$$\beta_{13} = \frac{2(1+\frac{1}{\chi})|eB|[2(\omega+\mathrm{i}\Gamma)^2-2\zeta+\xi-|eB|+2|eB|]}{(2\zeta-\xi+|eB|-2|eB|)^2},$$

$$\beta_{14} = \frac{8(1+\frac{1}{\chi})|eB|^2}{(2\zeta-\xi+|eB|-2|eB|)^2},$$

$$\beta_{15} = \frac{(1+\frac{1}{\chi})(\omega+\mathrm{i}\Gamma)^2}{2\zeta-\xi+|eB|-2|eB|},$$

$$\gamma_1 = \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta+\xi-2|eB|]}{(-2\zeta+2|eB|)^2} - \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta+2\xi-2|eB|]}{(-2\zeta-2\xi+2|eB|)^2},$$

$$\gamma_2 = \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi-|eB|}{-2\zeta-2\xi+2|eB|} - \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi-|eB|}{-2\zeta+2|eB|},$$

$$\gamma_3 = \frac{8|eB|^2}{(-2\zeta+2|eB|)^2} - \frac{8|eB|^2}{(-2\zeta-2\xi+2|eB|)^2},$$

$$\gamma_4 = \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta-\xi-2|eB|]}{(-2\zeta+2|eB|)^2} - \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta-2\xi-2|eB|]}{(-2\zeta+2\xi+2|eB|)^2},$$

$$\gamma_5 = \frac{8|eB|^2}{(-2\zeta+2|eB|)^2} - \frac{8|eB|^2}{(-2\zeta+2\xi+2|eB|)^2},$$

$$\gamma_6 = \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{-2\zeta+2\xi+2|eB|} - \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta-\xi-|eB|}{-2\zeta+2|eB|},$$

$$\gamma_7 = \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-2\zeta-\xi-2|eB|]}{(2\zeta+2|eB|)^2} - \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-2\zeta-2\xi-2|eB|]}{(2\zeta+2\xi+2|eB|)^2},$$

$$\gamma_8 = \frac{8|eB|^2}{(2\zeta+2|eB|)^2} - \frac{8|eB|^2}{(2\zeta+2\xi+2|eB|)^2},$$

$$\gamma_9 = \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2\zeta+2\xi+2|eB|} - \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta-\xi-|eB|}{2\zeta+2|eB|},$$

$$\gamma_{10} = \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-2\zeta+\xi-2|eB|]}{(2\zeta+2|eB|)^2} - \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-2\zeta+2\xi-2|eB|]}{(2\zeta-2\xi+2|eB|)^2},$$

$$\gamma_{11} = \frac{8|eB|^2}{(2\zeta+2|eB|)^2} - \frac{8|eB|^2}{(2\zeta-2\xi+2|eB|)^2},$$

$$\gamma_{12} = \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2\zeta-2\xi+2|eB|} - \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)-\zeta+\xi-|eB|}{2\zeta+2|eB|},$$

$$\gamma_{13} = \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta+2\xi+2|eB|]}{(-2\zeta-2\xi-2|eB|)^2} - \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta+\xi+2|eB|]}{(-2\zeta-2|eB|)^2},$$

$$\gamma_{14} = \frac{8|eB|^2}{(-2\zeta-2\xi-2|eB|)^2} - \frac{8|eB|^2}{(-2\zeta-2|eB|)^2},$$

$$\gamma_{15} = \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{-2\zeta-2|eB|} - \frac{(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+\zeta+\xi+|eB|}{-2\zeta-2\xi-2|eB|},$$

$$\gamma_{16} = \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta-2\xi+2|eB|]}{(-2\zeta+2\xi-2|eB|)^2} - \frac{4|eB|[(\omega+\mathrm{i}\Gamma)^2-(10\lambda_{\mathrm{SO}}^2+2\lambda_{\mathrm{R}}^2)+2\zeta-\xi+2|eB|]}{(-2\zeta-2|eB|)^2},$$

$$\gamma_{17} = \frac{8|eB|^2}{(-2\zeta+2\xi-2|eB|)^2} - \frac{8|eB|^2}{(-2\zeta-2|eB|)^2},$$

$$\gamma_{18} = \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta - \xi + |eB|}{-2\zeta - 2|eB|} - \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta - \xi + |eB|}{-2\zeta + 2\xi - 2|eB|},$$

$$\gamma_{19} = \frac{4|eB|[(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - 2\zeta - 2\xi + 2|eB|]}{(2\zeta + 2\xi - 2|eB|)^2} - \frac{2|eB|[(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - 2\zeta - \xi + 2|eB|]}{(2\zeta - 2|eB|)^2},$$

$$\gamma_{20} = \frac{8|eB|^2}{(2\zeta + 2\xi - 2|eB|)^2} - \frac{8|eB|^2}{(2\zeta - 2|eB|)^2},$$

$$\gamma_{21} = \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta - \xi + |eB|}{2\zeta - 2|eB|} - \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) + \zeta - \xi + |eB|}{2\zeta + 2\xi - 2|eB|},$$

$$\gamma_{22} = \frac{4|eB|[(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - 2\zeta + 2\xi + 2|eB|]}{(2\zeta - 2\xi - 2|eB|)^2} - \frac{4|eB|[(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - 2\zeta + \xi + 2|eB|]}{(2\zeta - 2|eB|)^2},$$

$$\gamma_{23} = \frac{8|eB|^2}{(2\zeta - 2\xi - 2|eB|)^2} - \frac{8|eB|^2}{(2\zeta - 2|eB|)^2},$$

$$\gamma_{24} = \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta + \xi + |eB|}{2\zeta - 2|eB|} - \frac{(\omega + i\Gamma)^2 - (10\lambda_{SO}^2 + 2\lambda_R^2) - \zeta + \xi + |eB|}{2\zeta - 2\xi - 2|eB|}.$$

Equations (B.2) and (B.6) are further rewritten as equations (15) and (16).

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